23/11/2015 الاثنين م. مجال 5 محاضرة

Binomial Random Variable

$$p(x) = \binom{n}{x} p^x q^{n-x} \quad , \quad x = 0, 1, 2, \cdots, n$$

x : is a binomial random variable that represent the no. of S's in n-trials.

n : No. of trials → success

p : probability of success in a single trial

q : $1-p$: probability of fail

Ex:-

In an Experiment 10% of people passes . suppose that 4 persons are selected at random.

a) find the probability that none of the four persons passes the test

b) find the probability that three of the four persons passes the test

derive a formula for $p(x)$, the probability distribution fn of the binomial Rariable x

Solution :-

none passes $p(FFFF)$ ≠

$$p(x) = {}^{n}C_x \, p^x \, q^{n-x}$$

at $x=0$, $n=4$, $p=0.1$, $q=0.9$

a) $\therefore p(0) = {}^{4}C_0 \, (0.1)^0 \, (0.9)^{4-0} = (0.9)^4$ #

b) $p(3) = {}^{4}C_3 \, (0.1)^3 \, (0.9)^{4-3} = 4(0.1)^3(0.9)$ #

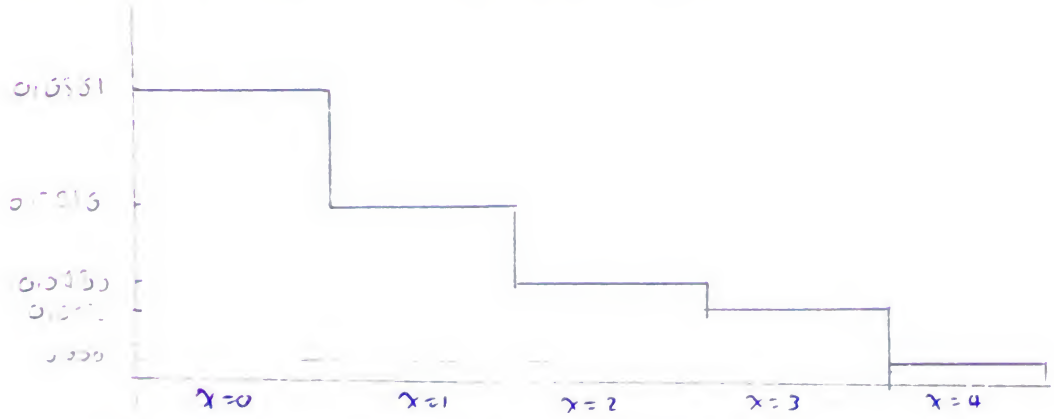
c)

x	0	1	2	3	4
$P(x)$	0.6561	0.2916	0.0486	0.0036	0.0001

~~P(0) =~~

$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$, $n = 4$, $p = 0.1$, $q = 0.9$

$P(0) = 0.6561$, $P(1) = {}^{4}C_{1}\,(0.1)^{1}\,(0.9)^{3} = 0.2916$ ...



$\Rightarrow$ for Binomial Random Variable

$\mu = np$;

$Var = npq$;

$S.D = \sigma = \sqrt{Var} = \sqrt{npq}$.

Sheet 5

3) prove that for any random variable x

i) $E(ax+b) = a\,E(x) + b$

$$E(ax+b) = \int_{-\infty}^{\infty} (ax+b)\,p(x)\,dx = a\int_{-\infty}^{\infty} x\,p(x)\,dx + b\int_{-\infty}^{\infty} p(x)\,dx$$

$\int_{-\infty}^{\infty} p(x)\,dx = 1$

$$E(ax+b) = a\,E(x) + b$$

7) Give a formula for $p(x)$ for a binomial random variable with $n=7$ and $p=0.2$

Solution:

$$p(x) = \binom{7}{x} (0.2)^x (1-0.2)^{7-x}$$

$$p(x) = \binom{7}{x} (0.2)^x (0.8)^{7-x}$$

8) Consider the following

$$p(x) = \binom{5}{x} (0.7)^x (0.3)^{5-x} \quad , x = 0,1,2,3,4,5$$

a) $n = ?$, b) $p = ?$

c) graph $p(x)$

d) find μ, $\underbrace{\sigma^2}_{\text{Var}}$, σ

Solution:

$n = 5$, $p = 0.7$, $q = 0.3$

x	0	1	2	3	4	5
$p(x)$						

$\mu = np = 5 \times 0.7 = 3.5$

$Var = npq = 5 \times 0.7 \times 0.3 =$

$SD : \sqrt{Var} =$

problem 10 : Report

11) A fair coin ⟶ 6 times . head ⟶ success
find

(i) probability that exactly 2 heads occur
(ii) probability that at least 4 heads
(iii) probability that no heads
(iv) probability that at least one head.

Solution

$n = 6$, $p = q = 0.5$

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

(i) $P(2) = {}^{6}C_{2}\,(0.5)^{2}\,(0.5)^{4} =$

(ii) $x = 4, 5, 6$

$P(4; 6, 0.5) + P(5; 6, 0.5) + P(6; 6, 0.5) =$

(iii) $P(0) = {}^{6}C_{0}\,(0.5)^{0}\,(0.5)^{6} =$

(iv) $x = 1, 2, 3, 4, 5, 6$

$= 1 - P(0; 6, 0.5)$